COMPUTATION OF THE FRACTAL DIMENSION OF METEOROLOGICAL QUANTITIES

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Abstract: *The aim of this preliminary study was to examine the capability of the fractal property's analysis to describe and explain the changes in microclimate and small water cycle which is violated by human activities. The research data from the fifteen measuring centres in the landscape are collected into the database and the fractal dimension of this data is calculated. For calculating of the fractal dimension is used the box counting method prelusively. Comparison of the values of fractal dimension of the data, e.g., from different kind of surface confirms the discriminatory capability of the fractal analysis. Moreover this method is applicable to the data correctness check.*

Keywords: *fractal, chaos, deterministic chaos, fractal dimension, box counting, time series.*

1 Introduction

1.1 Fractals

The fractals are an ideal mathematical objects postulated by Benoit Mandelbrot [1]. The key property of the fractals is their **self-similarity**. The self-similarity means, that the same object seems identical (or similar) in different scale of view. E.g., umbel, bracken, lungs, coastline and mountains from natural domain or von Koch snowflake, Sierpinski triangle and Cantor set from mathematical domain. As a fractals are usually considered geometrical objects. From this conception rise up the definition of fractal which tells that the fractal is an object which have Hausdorf dimension higher than topological dimension. At the other side the fractal characteristics can be manifested also on non-geometric object like a time series. For measuring rate of fractal characteristic of the given object is possible to use the Fractal Dimension.

1.2 Analysed object

The data for the fractal analysis are collected from the fifteen measuring centres (Figure 1) which are situated on the different places in South Bohemia around Třeboň town. Each of this centres measures 13 meteorological variables like a temperature in 2 metres and 30 centimetres above surface, humidity (in the same altitude), solar radiation and the radiation reflected from earth surface, rainfall, wind speed and direction and temperature and humidity of the earth in different depth. The data from these measuring centres are saved into the database with 5 minute period. The measuring points are categorized into group according to surface (meadow, tideland, pond, field, concrete surface etc.). The Třeboň region is well known for their ponds and it is surprising that this land has a problem with dehumidifying and dehydrating. The fractal analysis is another way how to analyse the measured data and gain the additional information.

Figure 1 One of the measuring centres.

2 Calculation of the Fractal Dimension (FD)

Fractal dimension is measure of fractality and is postulated on the example of the measuring of the coastline length (two dimensional object). Let consider a gauge for a measuring and this gauge have a given size. And if we use smaller gauge that our length of coastline will be longer (and vice versa). And with increasing seashore's segmentation increase measured length of coastline. In other words … when measure at a given spatial scale d, the total length of a crooked coastline L(d) is estimated as a set of N straight-line segments of length d. Because small details of the coastline not recognized at lower spatial resolutions become apparent at higher spatial resolutions, the measured length L(d) increases as the scale of measurement d increases. [7]. The Fractal dimension (FD) postulated by Felix Hausdorff and developed by Abram Samoilovitch Besicovitch is based on this idea and is defined as:

$$D = \frac{\log N}{\log(1/r)} \quad (1)$$

Where:

D … the fractal dimension

N… the number of an object's internal homotheties

r … the ratio of homothety

2.1 Box Counting Method

The formula (1) is for the ideal (mathematical) object. The Box Counting is method to estimating the Fractal Dimension of the real object – in this case time series. This method is also based on the coastline measuring principle. The estimated FD is an angular coefficient of the line which approximates the dependence of logarithm of minimal number of n-dimensional cubes (or sphere) which can cover the object and logarithm of reciprocal value of size of the n-dimensional cubes (or sphere) – formula (2). The Box Counting Method is described, e.g., in [3], [4], [5], [6], [7] and is illustrated on Figure 2.

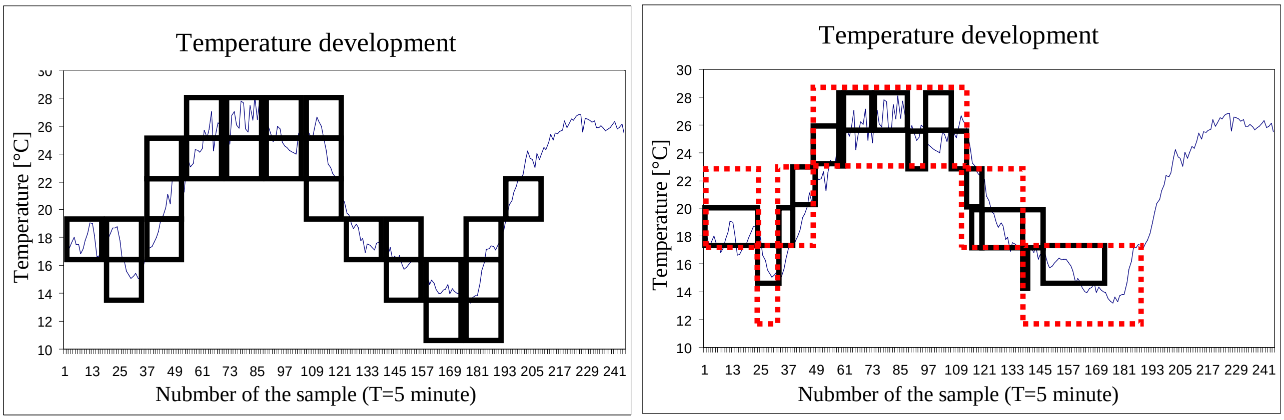
$$D = \lim_{\varepsilon \to 0} \frac{\log[C(\varepsilon)]}{\log \frac{1}{\varepsilon}} \quad (2)$$

Where:

D … the estimation of FD

ε … the size of the n-dimensional cubes

C(ε) … the minimal number of n-dimensional cubes



a) b)

Figure 2 Box counting for two and one-dimensional objects.

Since we calculate the fractal dimension of the one-dimensional series we work in one-dimensional space. Therefore also covering n-dimensional cube is one-dimensional – ergo abscissa. If we want to calculate the FD of the time series like a Figure we must to specify the time-scale for transformation (display) the time to a space. And FD is influenced by this given time-scale (especially by the rate between scale on the axis x and y).

Procedure of FD solving is consequential:

1) Set the minimal and maximal meaningful size of one-dimensional cube. In this moment is necessary prescribing the fixed rules to assess the minimal and maximal size of abscissa. The results depend on these values. It is well indicate on the Figure 3. The range of abscisses's size is need to choice from the part which is closed to linear (illustrated on the Figure 3 in the rectangular). But this graph is known from step four.
2) Cover the entire object by the abscissas for a given size (Figure 2b). Size is between min and max value defined in step 1.
3) Count number of abscissas required to cover the object and repeat step 2 and 3 for a different size of abscissas.
4) Draw the points of dependency (Figure 4).
5) Realise linear regression (Figure 4).
6) Calculate the coefficients of this line – the firs coefficient is angular coefficient, ergo estimated Fractal Dimension (Figure 4).

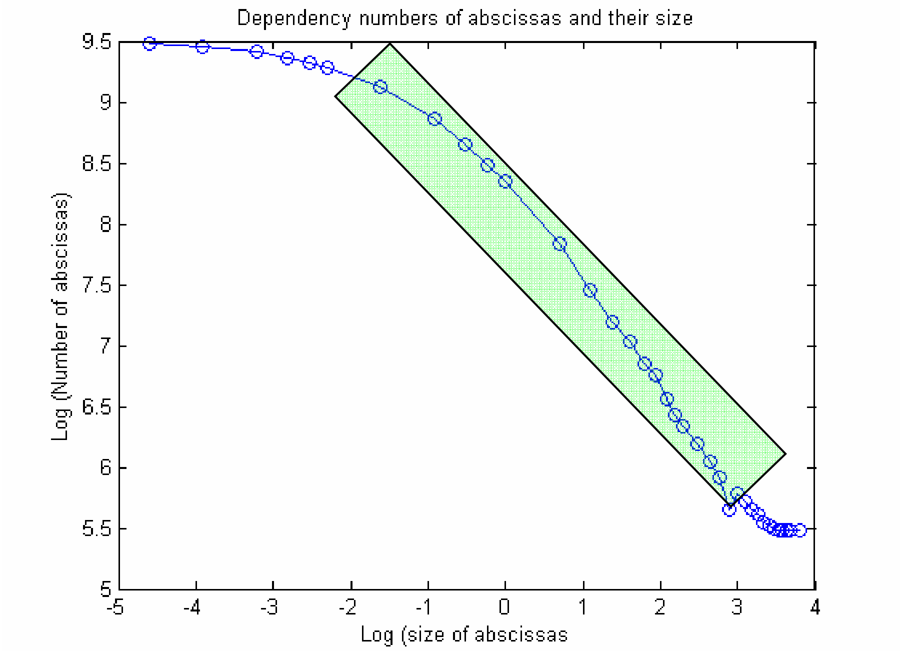


Figure 3 Influence of the abscissa's size to the estimation of the fractal dimension.

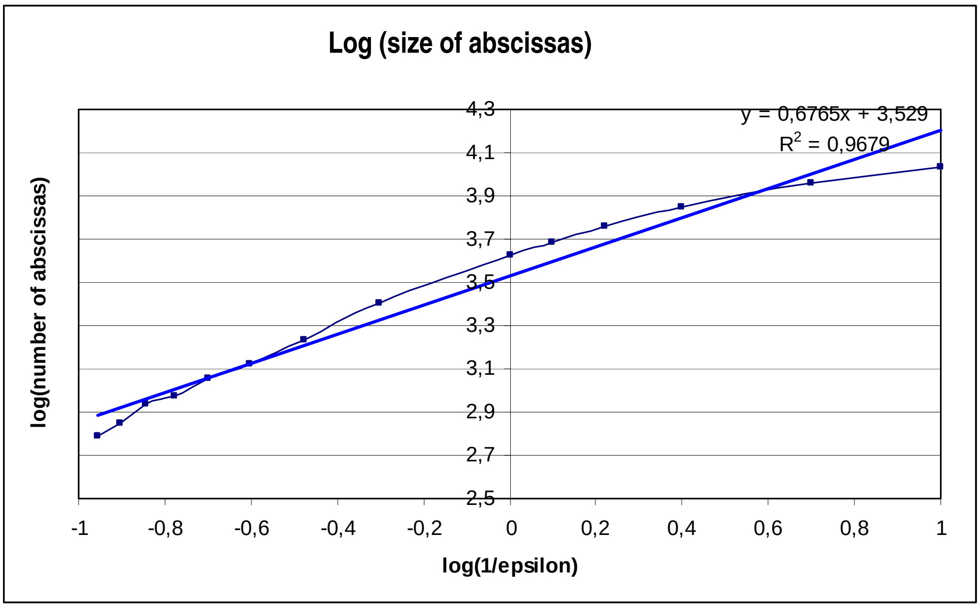


Figure 4 Calculating FD by Box Counting Method.

3 Results

The computation of Fractal Dimension has minimally two useful applications – Fractal analysis of ecosystem functions and check of the collected data correctness.

3.1. Fractal analysis of meteorological quantities

The goal of the project (referred in [9]) is among others concentrated on the development of biodiversity in the ecosystem. The research is oriented especially to micro-meteorology, to the thermodynamics of atmosphere in a close distance to surfaces of different localities and to the principles of Small Water Cycle SWC [2]. The identified value of the fractal dimension of the meteorological quantities can relate to development of biodiversity and violation of the SWC at the specific type of surface. This hypothesis is based on known relation between chaotic meteorological signals and fractal dimension and deterministic chaos principles. However verification of this hypothesis is not goal of this submission. The results of calculating are numerical expression of the FD's estimation of the different meteorological quantities at different places and in a different time.

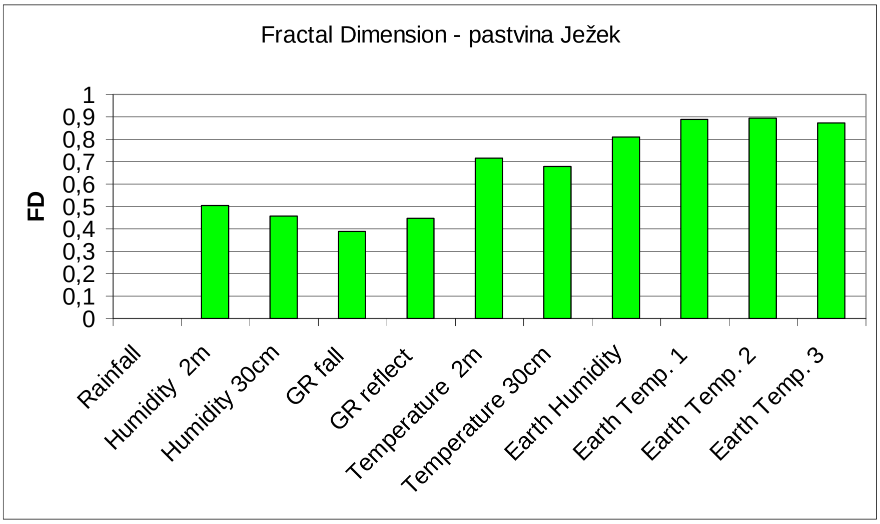


Figure 5 Fractal Dimension of different meteorological quantities at the Pastvina Ježek place on 2008 July.

On the Figure 5 is shown that value of the FD of different quantities is also different. Lower value of the solar radiation's FD is expected, because the segmentation of this quantity is a little. But higher value of Earth temperatures than temperature at 2 m and 30 cm is surprise.

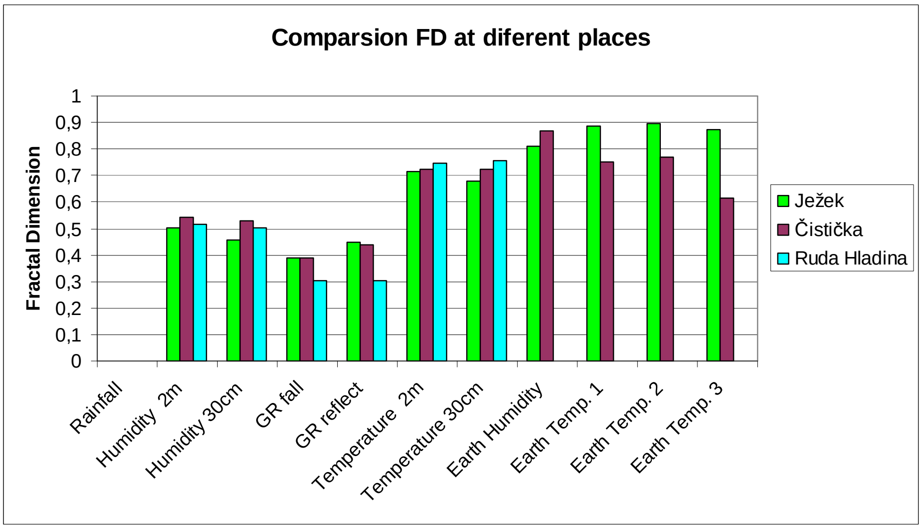


Figure 6 Fractal Dimension of meteorological quantities at the three different places (meadow, concrete, water surface) on 2008 July.

On the Figure 6 is shown that the differences between values of FD at various places are lower than differences between values of FD of various quantities. The value of the fractal dimension of earth temperature is significantly higher on the green surface than on the concrete surface. And finally the value of FD is relatively stable over a year as it shown at the Figure 7.

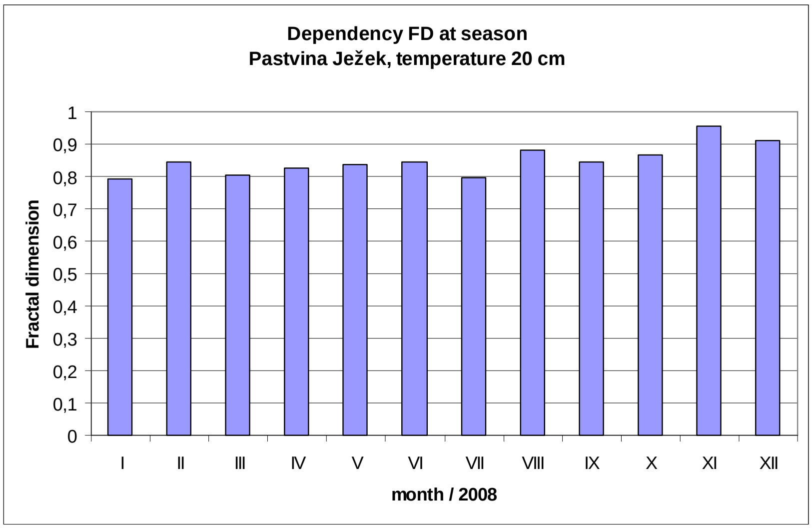


Figure 7 Fractal Dimension of the temperature in 20 cm above surface at the Pastvina Ježek (meadow) on 2008 July.

3.2 Correctness of collected data

Next use of the fractal analysis is to check the correctness of collected date. The data from the remote measuring centres are saved in the database of the TokEnLek server. Any amount of the data make impossible manually checks them and probability of the emergence of the error is not zero. The fractal analysis is another tool to identify this error. As it shown at the Figure 8 the Fractal analysis is sensitive to data incoherence. The certain fractal dimension is inherent to the given fractal object and the change of FD denotes some change of this object.

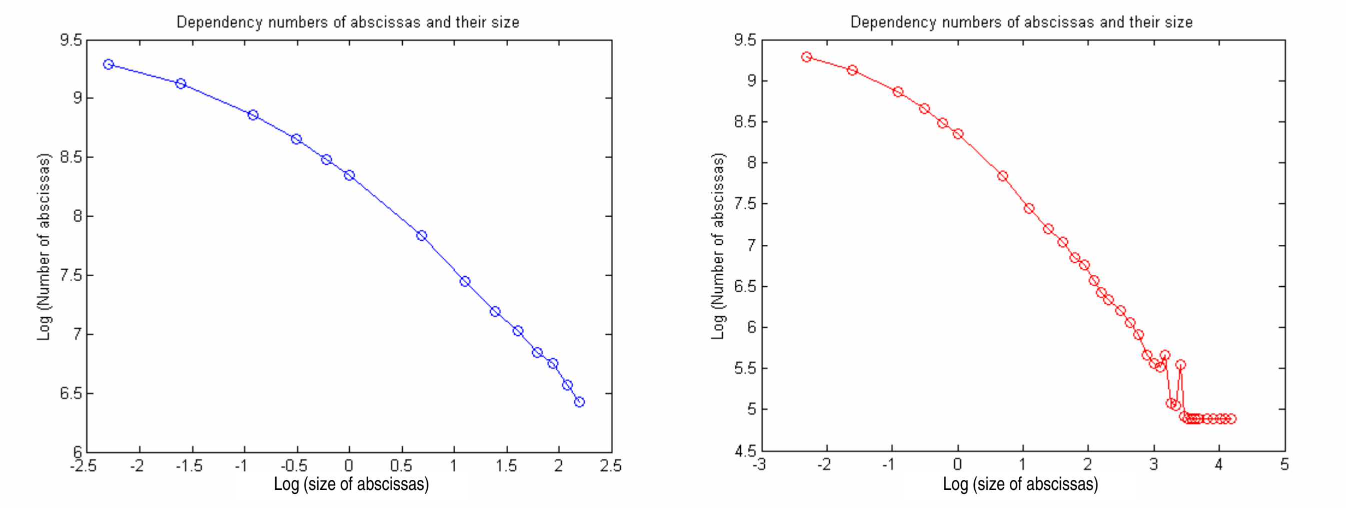


Figure 8 Sensitivity of the Fractal analysis to incoherent data. To the time series (right part) is added incorect value.

4 Conclusion

The experimental results show that the fractal dimension of micro-meteorological quantities has a relation with the environmental factors. E.g., fractal dimension of the earth temperature under the concrete surface (where the violation of the SWC is expected) is significantly lower than a fractal dimension from pasture and above the earth surface vice versa. These results will need the interpretation from the ecosystem's point of view, but there is a clear discriminatory capability of this method. Moreover this fractal analysis is applicable to the data correctness check. The method of determining the value of the fractal dimension was tested from the methodological point of view and it is very sensitive to the choice of suitable range of the size of covering n-dimensional cube, but the reliability of measuring is possible to reach out by an assignment and keep perspicuous rules this choice.

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